Data Structures and Algorithms

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1 Pennies

You have 27 pennies, 26 weight the same, one weighs less. you also have a judges scale. Find the one that weighs less? How many measurements do you need to make?

2 Optimizing a simple problem

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000. Give a

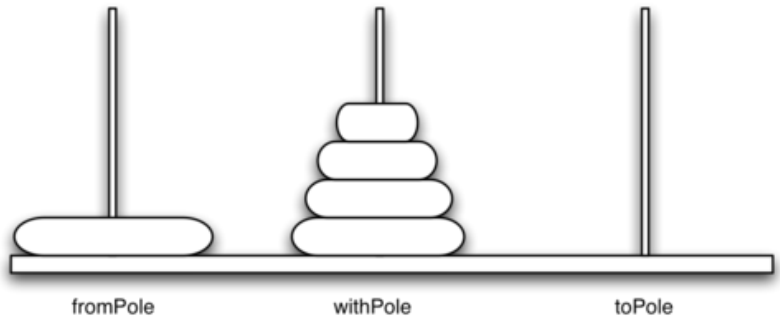
a) linear

b) constant

algorithm written in pseudocode.

3 Towers of Hanoi

There are three rods and a number of disks of different sizes, which can slide onto any rod. The puzzle starts with the disks in a neat stack in ascending order of size on one rod, the smallest at the top, thus making a conical shape.



The objective of the puzzle is to move the entire stack to another rod, obeying the following simple rules:

Only one disk can be moved at a time. Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod. No larger disk may be placed on top of a smaller disk.

Given n disks, what is the minimal number of moves required to solve a Tower of Hanoi? Prove your answer!

4 Comparing integers

Sort array of 5 integers with a max of 7 compares!

5 Searching for a sum in an array

Write a program that, given an array $A[]$ of n numbers and another number x, determines whether or not there exist two elements in S whose sum is exactly x. What is the computational-complexity of your solution?

6 Median of two sorted arrays of same size

There are 2 sorted arrays A and B of size n each. Write an algorithm to find the median of the array obtained after merging the above 2 arrays(i.e. array of length $2n$). The complexity should be $\mathcal{O}(log(n))$.

7 Asymptotic notation aka. Big $\mathcal{O}$ notation

True or false? If no, provide a counter-example! Justify your answer!

- $f(n) = \Theta(n) \wedge g(n) = \Omega(n) \implies f(n)g(n) = \Omega(n^2)$
- $f(n) = \Theta(1) \implies n^{f(n)} = \mathcal{O}(n)$
- $f(n) = \Omega(n) \wedge g(n) = \mathcal{O}(n^2) \implies g(n)/f(n) = \mathcal{O}(n)$
- $f(n) = \mathcal{O}(n^2) \wedge g(n) = \mathcal{O}(n) \implies f(g(n)) = \mathcal{O}(n^3)$
- $f = \mathcal{O}(log(n)) \implies 2^{f(n)} = \mathcal{O}(n)$
- $f(n) = \Omega(log(n)) \implies 2^{f(n)} = \Omega(n)$